

# Experimental demonstration of wave-particle duality relation based on coherence measure

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Wave-particle duality is a typical example of Bohr's principle of complementarity that plays a significant role in quantum mechanics. Previous studies used visibility to quantify wave property and used path information to quantify particle property. However, coherence is the core and basis of the interference phenomena of wave. If we use it to characterize wave property, which will be useful to strengthen the understanding of wave-particle duality. A recent theoretical work [Phys. Rev. Lett. 116, 160406 (2016)] found two relations between wave property quantified by coherence in different measure and particle property. Here, we demonstrated the wave-particle duality based on two coherence measures quantitatively for the first time. The path information can be obtained by the discrimination of detector states encoded in polarization of photons corresponding each path and mutual information between detector states and the outcome of the measurement performed on them. We obtained wave property quantified by coherence in  $l_1$  measure and relative entropy measure using tomography of photon state that encoded in paths. Our work will deepen people's further understanding of coherence and provides a new angle of view for wave-particle duality.

**Introduction.**—Bohr's principle of complementarity is at the heart of quantum mechanics. The core of this principle is that an object has multiple properties and that the observation of different properties requires different methods. This means that some properties cannot be optimally measured simultaneously even in principle [1, 2]. A typical example is wave-particle duality. A particle that goes through an interferometer can exhibit either wave or particle properties. The particle properties are characterized by information about which path the particle took, while the wave properties are characterized by the visibility of the interference pattern. If we have complete information about which path the particle took, the visibility is zero, and if the visibility is at its maximum value of one, we have no information about which path the particle took. There are, of course, intermediate situations in which some path information leads to a reduction in the visibility, and this has been studied experimentally [3–6]. The intermediate case was first investigated by Wootters and Zurek in 1978 [7]. In 1988, Greenberger and Yasin found an inequality that expresses the tradeoff between visibility and path information [8]

$$V^2 + D^2 \leq 1, \quad (1)$$

where  $V$  is the visibility and  $D$  is a measure of the path information. The path information in their case is a result of the fact that due to the preparation of the particle state inside the interferometer (for example, due to an unbalanced initial beam splitter), the particle has different probabilities to take different paths. If the particle is equally likely to take each of the paths, one has no path information, but if one path is highly probable, one has a large amount of path information. Englert, in a seminal paper, added detectors to the scenario, and also obtained a relation of the form given in Eq. (1), though now  $D$  corresponds to a different quantity than previously [9].

Different states of the detectors correspond to different paths, and Englert's measure of path information,  $D$ , is based on one's ability to distinguish the different detector states. This ability includes the information about the preparation of the particle, if one path is more likely than another this affects the discrimination probabilities, but it also includes information gleaned from measuring the detectors. The inequality given in Eq. (1) has been demonstrated experimentally in atoms [10], nuclear magnetic resonance [11, 12], by using a faint laser [13], and by using single photons in a delayed-choice scheme [14]. Besides two-path case, many theoretical and experimental studies of path-visibility relation in multibeam interferometer have been proposed in Refs. [15–19]. The visibility of the interference pattern is usually used to quantify the wave nature of the particles, but coherence is the core and basis of wave. If we use quantified coherence to characterize wave property, which will be useful to further understand wave-particle duality better.

Recently two measures of quantum coherence have been proposed [20]. One is based on the  $l_1$  norm of the off-diagonal elements of the density matrix in the path basis, and the other is an entropic measure. These measures were the result of a resource theory of quantum coherence, and this theory has stimulated further studies [21–26]. The first application of the recently defined coherence measures to wave-particle duality relations was by Bera, *et al.*, which used the  $l_1$  coherence measure to quantify the wave nature of the particle [27]. The path information was characterized by an upper bound to the probability of successfully discriminating the detector states by means of unambiguous state discrimination. Subsequently, two new duality relations were found [28]. The first used the  $l_1$  coherence measure for the wave properties and the probability of successfully discriminating the detector states using minimum-error state discrimination for the path information.

The second used the entropic measure of coherence for the wave properties and the mutual information between the detector states and the measurement used to distinguish them for the path information. Up to now these wave-particle duality relations have not been demonstrated experimentally.

In this work we firstly report an experimental demonstration of wave-particle duality relation based on coherence measures [28]. Here, the coherence and path information of photons are studied in Mach-Zehnder interferometer. The detector states are encoded in polarization freedom of photons, we acquire particle property characterised by path information using minimum-error state discrimination between detector states and mutual information between detector states and the outcome of the measurement performed on them. The photon state is encoded in paths freedom and observed coherence using tomography of photon state.

The first relation between coherence and path information is expressed as [28]

$$(P_s - \frac{1}{N})^2 + X^2 \leq (1 - \frac{1}{N})^2. \quad (2)$$

Here,  $(P_s - 1/N)$  represents particle property, and  $X$  represents wave property.  $P_s$  is the average probability of successfully discriminating detector states which proposed by Englert [9]. According to his model, a detector is introduced in each path and it coupled to the path. We can get path information by discrimination of detector states. If the detector states are orthogonal, full which-way information can be obtained. If the detector states are non-orthogonal, partial which-way information can be obtained.  $N$  is the number of paths, and  $N = 2$  in our experiment. If we have no prior information about path that particle passed through, the probability that we guess correctly is  $1/N$ . Thus  $P_s - 1/N$  can be understood as the measure of how much better we can do by using detectors than by just guessing.  $X$  can be written as  $X = (1/N)C_{l_1}(\rho)$ , where  $C_{l_1}(\rho) = \sum_{i,j=1, i \neq j}^N |\rho_{ij}|$ . Considering a photon entering a Mach-Zehnder interferometer, the photon in the superposition state inside the interferometer. The photon state is encoded in path freedom of photons and expressed as  $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$ , the orthonormal basis states  $|1\rangle, |2\rangle$  correspond to the path-1 and path-2. As shown in Fig. 1, the photon state prepared by the single photons through a polarizing beam splitter (PBS), a half-wave plate (HWP) HWP1 and the beam displacer (BD) BD1 which makes the vertical polarized component transmit directly but makes the horizontal polarized component undergo a 4mm lateral displacement. To detect path information, path states are coupled to two detector states encoded in the polarization freedom of the photons. One detector in each path and detector states are expressed as  $|\eta_1\rangle, |\eta_2\rangle$

$$\begin{aligned} |\eta_1\rangle &= \cos \vartheta |H\rangle + \sin \vartheta |V\rangle, \\ |\eta_2\rangle &= \cos \vartheta |H\rangle - \sin \vartheta |V\rangle. \end{aligned} \quad (3)$$

The two detector states are encoded in horizontal polarization  $|H\rangle$  and vertical polarization  $|V\rangle$ , prepared by rotating

HWP2 and HWP3. Different detector states are realized by wave plates with different angles  $\vartheta$ . After the photon passed through the BD1 and interacted with the detector system, the state of the entire system becomes

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle|\eta_1\rangle + |2\rangle|\eta_2\rangle). \quad (4)$$

We can get the photon density matrix by tracing out the detector,

$$\rho = Tr_{det}(|\Psi\rangle\langle\Psi|) = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}\cos 2\vartheta \\ \frac{1}{2}\cos 2\vartheta & \frac{1}{2} \end{pmatrix}. \quad (5)$$

Thus we obtain coherence measure  $X$  is

$$X = \frac{1}{2}\cos 2\vartheta. \quad (6)$$

This is what we need to measure in the experiment. The experimental realization of observing coherence  $X$  is shown in measurement part of Fig. 1. We set HWP4 at  $0^\circ$  and observe coherence by achieving the tomography of particle state. Let the horizontal polarization component of photons in path-1 and path-2 overlap, and so it is with vertical polarization component in path-1 and path-2. Then we perform tomography separately and get two density matrixes. The density matrixes of photon state can be expressed as the sum of weights of two density matrixes.

Because path information is encoded in detector states and we need to discriminate them, we also need to know detector density matrix by tracing out photon states,

$$\rho_{det} = Tr_{photon}(|\Psi\rangle\langle\Psi|) = \frac{1}{2}(|\eta_1\rangle\langle\eta_1| + |\eta_2\rangle\langle\eta_2|). \quad (7)$$

In order to measure the distinguishability of paths thereby obtain path information,  $|\eta_1\rangle$  and  $|\eta_2\rangle$  need to be discriminated. Here, we adopt the method of minimum-error state discrimination which always can get a result but it may be wrong [29], though the probability of making an error is minimized. So we find that the optimized measurements maximize the probability of successfully identifying the detector states  $P_s$ . As for detector states that we choose  $|\eta_1\rangle$  and  $|\eta_2\rangle$ , the optimal measurement is a projective measurement onto the states [30]

$$\begin{aligned} |\phi_1\rangle &= \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \\ |\phi_2\rangle &= \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle). \end{aligned} \quad (8)$$

Thus, the optimal POVM elements are  $\Pi_1 = |\phi_1\rangle\langle\phi_1|$  and  $\Pi_2 = |\phi_2\rangle\langle\phi_2|$ . The average probability of successfully identifying the detector states is

$$P_s = \sum_{i=1}^2 \frac{1}{2} \langle \eta_i | \Pi_i | \eta_i \rangle = \frac{1}{2} + \frac{1}{2} \sin 2\vartheta. \quad (9)$$

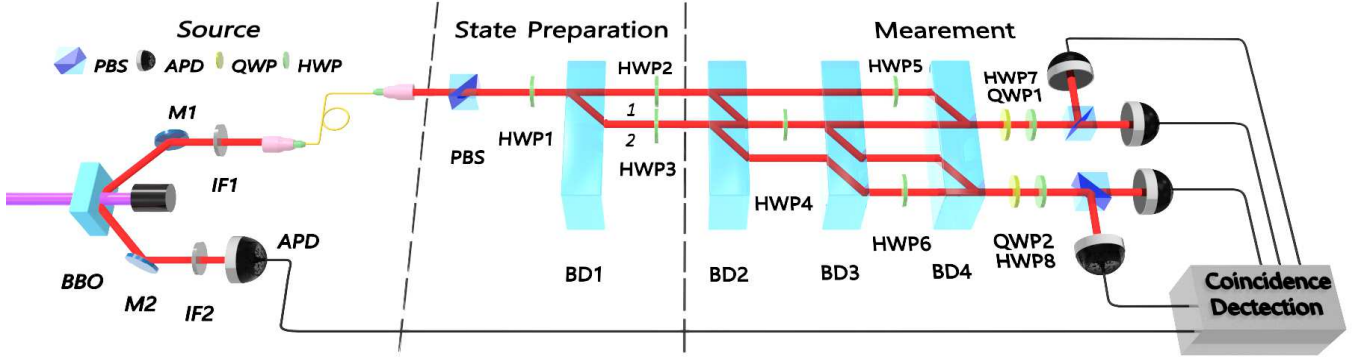


FIG. 1: Experimental setup. In state preparation part, Because the photon pass through path-1 and path-2 equiprobably, the angle of half-wave plate HWP1 is  $22.5^\circ$ . The detector states can be changed by rotating HWP2 and HWP3. We have measure coherence and path information independently. The HWP4 is  $0^\circ$  or  $45^\circ$ , up to we observe coherence by tomography of particle state or obtain path information in measurement part. The HWP5 and HWP6 both are rotated by  $45^\circ$ .

In light source part, as shown in Fig. 1, a laser at 404 nm wavelength pumps a type-II beamlike phase-matching beta-barium-borate (BBO,  $6.0 \times 6.0 \times 2.0 \text{ mm}^3$ ,  $\vartheta = 40.98^\circ$ ) crystal to produce the degenerate photon pairs. After being redirected by the mirrors (M1 and M2, as in the Fig.1) and passing through the interference filters (IF,  $\Delta\lambda=3 \text{ nm}$ ,  $\lambda=808 \text{ nm}$ ), the photon pairs generated in the spontaneous parametric down-conversion (SPDC) process are coupled into single-mode fibers separately. Single photon state is prepared by triggering on one of these two photons, and the total coincidence counting rate collected by the avalanche photo-diodes (APD) are about  $5 \times 10^3$  in one second. Coherence and path information are observed independently. To measure the probability of successfully identifying the detector states  $P_s$ , we set HWP4 at  $45^\circ$  and obtain path information by achieving the minimum-error state discrimination in measurement part of experimental setup. Photon passed through path-1 or path-2 determined by the results of discriminating detector states. If the detector state is found to be  $|\eta_1\rangle$ , the photon passed through path-1, otherwise it passed through path-2. The probabilities  $p(1, |\eta_1\rangle)$  and  $p(2, |\eta_2\rangle)$  of successfully identifying the detector state  $|\eta_1\rangle$  in path-1 and  $|\eta_2\rangle$  in path-2 are another quantity that we need to measure in experiment. The average probability of the successful discrimination is  $P_s = p(1, |\eta_1\rangle) + p(2, |\eta_2\rangle)$ . Optimal measurement is constructed in Eq. (8), and HWP7 and HWP8 just need to be rotated by  $22.5^\circ$ .

With measurement described above, we finally obtain coherence and path information for different detector states, as depicted in Fig. 2. For  $\vartheta = 0^\circ$ , no which-way information is stored while maximum coherence is observed and  $X_{\max} = 0.4997 \pm 0.0001$ . For  $\vartheta = 45^\circ$ , full which-way information is obtained while no coherence is observed and  $P_{\max} = 0.4991 \pm 0.0001$ . For intermediate values of  $\vartheta$ , incomplete which-way information is stored and partial coherence is observed. Each quantity varies from 0 to 0.5, and the increasing path information  $P$  leads to decreasing  $X$ , which is consis-

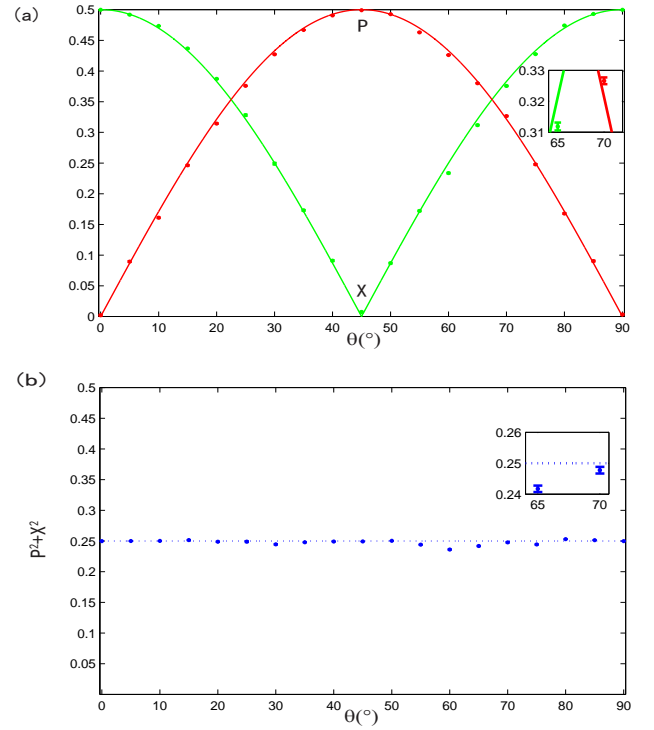


FIG. 2: Results for the relation between coherence in  $I_1$  measure and path information. Fig. 2(a) shows that coherence  $X$  (green) and distinguishability  $P$  (red) as a function of the detector states  $\vartheta$ . Here, we define  $P = P_s - 1/2$ . The solid lines are the theoretical expectations Eqs. (6) and (9). Fig. 2(b) shows the sum of the square of coherence and distinguishability. The blue dashed line is the theoretical values. Error values are too small to identify in the proportion of such coordinate, so we provide partial enlarged drawings with the magnifying power of 5x.

tent with the Bohr's principle of complementarity. From Fig. 2(b), we can see the sum of the square of  $X$  and square of  $P$  is invariant with respect to  $\vartheta$  and close to the upper bound in inequality Eq. (2).

The same experimental setup can be used to demonstrate the relation between coherence in relative entropy measure and path information in terms of mutual information [28],

$$C(\rho) + \text{Acc}(D) \leq H(p_i). \quad (10)$$

Where  $C$  represents the coherence in relative entropy measure. The relative entropy measure of coherence for a density matrix  $\rho$  is given by  $C(\rho) = S(\rho_{\text{diag}}) - S(\rho)$ , where  $\rho_{\text{diag}}$  is a diagonal density matrix of  $\rho$  and the von Neumann entropy is  $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$ .  $\rho$  is given by Eq. (5). Thus, the theoretical coherence value of photon in relative entropy measure  $C(\rho)$  is expressed as

$$C(\rho) = 1 + (\cos^2 \vartheta \log_2 \cos^2 \vartheta + \sin^2 \vartheta \log_2 \sin^2 \vartheta). \quad (11)$$

As for path information, we quantify it using the mutual information  $H(M : D)$  between the detector states labeling the paths and the results of probing them and  $\text{Acc}(D) = \max H(M : D)$  in Eq. (10) denotes maximum mutual information using the optimal POVM measurement. The  $H(M : D)$  can be expressed as  $H(M : D) = H(D) + H(M) - H(p_{ij})$ .  $D$  is a variable corresponding to the detector states  $|\eta_i\rangle$ , and  $i = 1, 2$ . From Eq. (7), the detector states  $\rho_i = |\eta_i\rangle\langle\eta_i|$  appear with probability  $p_i = 1/2$ . Its information content is  $H(D) = H(p_i) = 1$ , where  $H(p_i) = -\sum_{i=1}^2 p_i \log_2 p_i$  is the Shannon entropy. Then detector states are discriminated with optimal POVM  $\Pi_i$  to identify the path that photon passed through. The variable of measure result is expressed as  $M$ . If  $M = i$ , we think the photon pass through the path- $i$ . The probability of getting  $M = i$  is  $p(M = i) = \text{Tr}(\Pi_i \rho_{\text{det}})$ . According to Eq. (7) and Eq. (8), we get the information content is  $H(M) = H(q_i) = 1$ , there  $q_i \equiv p(M = i)$ . The joint distribution of the two variables  $D$  and  $M$  is written as  $p_{ij} \equiv p(M = i, D = j) = \text{Tr}(\Pi_i \rho_j) p_j$ . At last, the mutual information  $H(M : D)$  is expressed as

$$H(M : D) = 2 + 2p_{11} \log_2 p_{11} + 2p_{12} \log_2 p_{12}, \quad (12)$$

where  $p_{11} = p_{22} = \frac{1}{4} + \frac{1}{4} \sin 2\vartheta$  and  $p_{12} = p_{21} = \frac{1}{4} - \frac{1}{4} \sin 2\vartheta$ . If the two variables are perfectly correlation, the mutual information  $H(M : D) = H(D)$ . If not, the mutual information is equal to zero. The experimental coherence value of photon in relative entropy measure can be obtained by tomography of photon state in path. The final results are depicted in Fig. 3.

Fig. 3(a) shows that the experimental results of  $H$  and  $C$  agree well with theoretical predictions. For  $\vartheta = 0^\circ$ ,  $\vartheta = 45^\circ$ , and intermediate values  $\vartheta$ , the change trend of coherence and mutual information is the same as Fig. 2(a). The increasing path information  $H$  leads to decreasing  $C$ , each quantity varies from 0 to 1. This is also consistent with the Bohr's principle of complementarity. From Fig. 3(b), we can see the results in our experiment agree with the second inequality theoretical predictions Eq. (10). The sum of the  $C$  and  $H$  as a function of

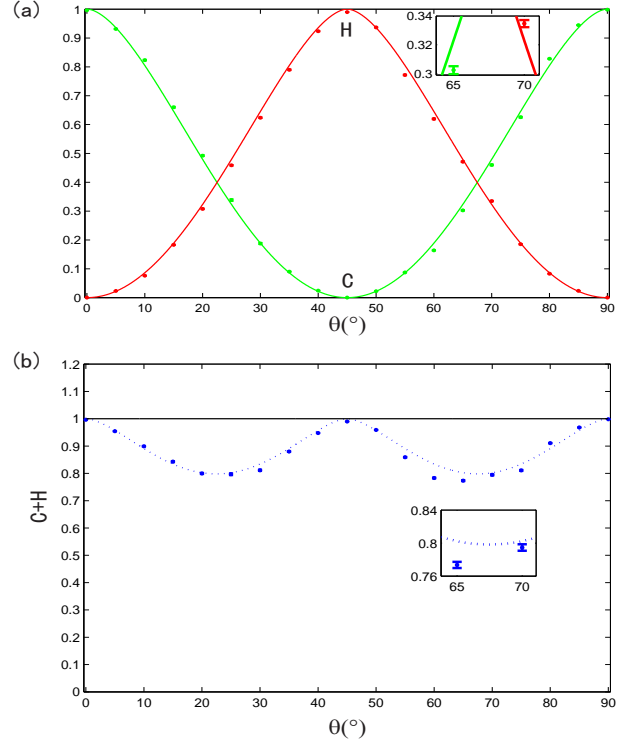


FIG. 3: Results for the relation between coherence in relative entropy measure and path information. Fig. 3(a) shows that coherence  $C$  (green) and the mutual information between the detector states labeling the paths and the results of probing them  $H$  (red) as a function of the detector states  $\vartheta$ . The solid lines are the theoretical expectations Eqs. (11) and (12). Fig. 3(b) shows the sum of coherence and the mutual information. The blue dashed line is the theoretical values. The black solid line represents the upper bound of inequality Eq. (10). Error values are too small to identify in the proportion of such coordinate, so we provide partial enlarged drawings with the magnifying power of 5x.

the detector states  $\vartheta$  and up to the upper bound in inequality Eq. (10) when  $\vartheta = 0^\circ$  and  $\vartheta = 45^\circ$ .

The statistical errors of experimental results are relatively small, since counting rate of source is high. The maximal error and minimum error are  $\pm 0.0042$ ,  $\pm 0.0001$  respectively. Then the errors in our experiment mainly stem from the inaccuracy of angles controlled by the wave plates and imperfect visibility.

**Conclusions.**—Previous research used different quantities to describe wave property in multipath interferometers, and tried to use other quantities as generalizations of the visibility. Coherence is the core of interference phenomenon, so quantified coherence as generalization of the visibility describes wave property is reasonable. In our experiment, the bound of inequality based on the  $l_1$  coherence measure can be reached with any kinds of detector states, this means that  $\vartheta$  can be any values. But the bound of inequality based on the entropy coherence measure can be reached with orthogonal detector states. Anyhow, the two relations between path information



and coherence in  $l_1$  and entropy measures Eq. (2), Eq. (10) both are tight, at least in the case  $N = 2$ . Our experimental results suggest that the quantified coherence in  $l_1$  norm and entropic measures are strong candidates with respect to generalization of the visibility describing wave property.

To summarize, we performed the first quantitative test of the recently proposed two wave-particle duality relations based on coherence measures in a Mach-Zehnder interferometer. In experiment, we obtained measurements of coherence and path information for different detector states applied to the interferometer. Our results match the theoretical predictions. The duality relations sustained the test in two-path interferometer. Our work can deepen people's further understanding of coherence and wave-particle duality relation.

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